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## VIBRAZIONI INDOTTE DALL'INFISSIONE DI PALI PREFABBRICATI

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### SOMMARIO

In ambito urbano la scelta progettuale dell'opera di fondazione è spesso condizionata dagli effetti che le tecnologie costruttive possono avere sugli edifici posti in prossimità dei cantieri.

In particolare l'infissione di pali e di palancole induce vibrazioni che si propagano nel terreno, generando spesso disturbi ed alcune volte danni nelle compagini strutturali particolarmente vulnerabili.

Nell'articolo si riportano e si analizzano criticamente i risultati di misure di vibrazione eseguite in siti di origine alluvionale, durante prove d'infissione di pali prefabbricati cilindrici e troncoconici con attrezzature di diversa energia.

L'elaborazione statistica dei dati ha consentito di calibrare, con riferimento ai terreni indagati, delle relazioni di pseudo-attenuazione per valutare l'andamento del moto vibratorio, in termini di velocità di picco delle particelle, al crescere della distanza dalla sorgente ed al variare dell'energia normalmente impiegata per l'infissione.

Vengono infine confrontati i valori di velocità rilevati con i limiti suggeriti dalle norme UNI, DIN e dalla normativa europea EC3 e suggeriti dei criteri su cui impostare la scelta delle attrezzature d'infissione, con riferimento ad alcune tipologie strutturali caratteristiche dell'ambiente urbano.

Parole chiave: pali infissi, vibrazioni, danni, normativa

### 1 INTRODUZIONE

Il problema delle vibrazioni, con riferimento al moltiplicarsi delle fonti connesse alle attività umane, ha assunto, negli ultimi anni, sempre maggiore importanza in relazione alle caratteristiche funzionali e strutturali degli edifici ed in relazione alla particolare vulnerabilità delle opere monumentali.

Sorgenti di vibrazione quali attività di cantiere, funzionamento di macchine, traffico stradale e ferroviario, possono essere origine di preoccupazione nei confronti delle costruzioni, in particolare nei casi in cui le opere manifestino stati di sofferenza con lesioni di vario tipo. In generale i danni strutturali attribuibili alle vibrazioni sono abbastanza rari, mentre sono più frequenti i cosiddetti danni "di soglia" che, senza compromettere la sicurezza, si manifestano sotto forma di fessure nell'intonaco, allargamenti di crepe già esistenti, danneggiamenti di elementi architettonici, cavillature di pavimenti per microoscillazione dei solai, con conseguenze di carattere estetico. Inoltre questo problema è particolarmente sentito nei riguardi della conservazione delle opere di pregio storico ed artistico.

In tale ambito assume particolare importanza la previsione degli effetti delle vibrazioni trasmesse durante l'infissione di pali prefabbricati in calcestruzzo, per una

corretta scelta dell'attrezzatura, maglio diesel o idraulico, e della relativa energia, in relazione alle caratteristiche del terreno e degli edifici posti in adiacenza.

### 2 VIBRAZIONI NEL TERRENO DERIVANTI DALLE OPERAZIONI D'INFISSIONE

Le onde derivanti da una sorgente costituita da un palo in fase d'infissione sono in genere onde di volume sia di compressione che di taglio e onde di superficie.

Le onde di compressione (P) e di taglio (S) si propagano dalla punta del palo con fronte d'onda sferico, mentre l'attrito laterale lungo il fusto origina onde di taglio (S) polarizzate verticalmente che si propagano con fronte d'onda conico (figura 1).

In corrispondenza della superficie parte dell'energia delle onde P ed S viene riflessa e parte dà origine ad onde di taglio con componente orizzontale e verticale, in particolare onde di Rayleigh (R).

L'ampiezza del moto particellare indotto dalle vibrazioni si attenua all'aumentare della distanza dalla sorgente.

Parte dell'attenuazione è dovuta alla distribuzione dell'energia su un fronte d'onda che si propaga con superficie sempre maggiore (*radiation damping*) e che può essere descritta dalla seguente equazione:

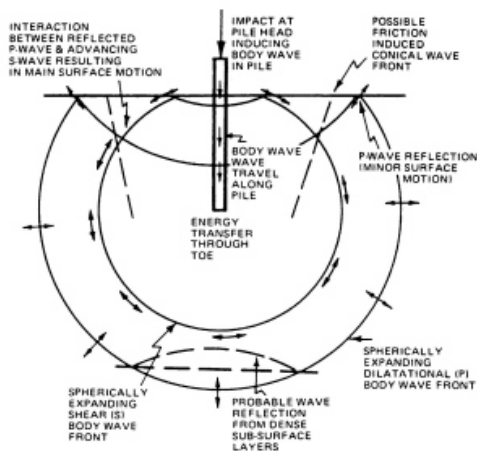


Figura 1. Propagazione di fronti d'onda durante l'infissione di un palo (Kim e Lee, 2000).

$$w = w_0 \left( \frac{r_0}{r} \right)^n \quad (1)$$

con  $w$  e  $w_0$  ampiezza della vibrazione a distanza rispettivamente  $r$  e  $r_0$  dalla sorgente e  $n$  coefficiente di attenuazione che dipende dal tipo d'onda e dal tipo di sorgente (tabella 1); parte dell'energia viene dissipata per attrito dal mezzo in cui l'onda si propaga (*material damping*). L'effetto combinato del radiation e del material damping può essere descritto dall'espressione (Bornitz, 1931):

$$w = w_0 \left( \frac{r_0}{r} \right)^n e^{-a(r_0-r)} \quad (2)$$

in cui con  $a$  si indica il coefficiente di attenuazione dovuto al material damping (tabella 2). Tale coefficiente è direttamente proporzionale alla frequenza ( $f$ ), al damping ratio ( $D$ ) ed inversamente proporzionale alla velocità di propagazione dell'onda ( $V$ ), secondo l'espressione:

$$a = \frac{2\pi f D}{V} \quad (3)$$

Tabella 1. Coefficiente di attenuazione per radiation damping (Kim and Lee, 1998).

Posizione delle sorgente	Tipo di sorgente	Tipo d'onda	n
superficiale	Puntiforme	di volume	2
		di superficie	0.5
superficiale	linea infinita	di volume	1
		di superficie	0
profonda	puntiforme linea infinita	di volume	1
			0.5

Tabella 2. Classificazione del terreno in base all'attenuazione per material damping (Woods, 1997).

Classe	$a \text{ (m}^{-1}\text{)}$		Numero di colpi prova SPT
	5 Hz	50 Hz	
I	0.01 – 0.03	0.1 – 0.3	$N_{SPT} < 5$
II	0.003 – 0.01	0.03 – 0.1	$5 < N_{SPT} < 15$
III	0.0003 – 0.003	0.003 – 0.03	$15 < N_{SPT} < 50$
IV	$< 0.0003$	$< 0.003$	$N_{SPT} > 50$

Con riferimento all'infissione di un palo, definibile come sorgente puntiforme, si può osservare come le onde di superficie subiscano un decadimento d'ampiezza per radiation damping proporzionale alla radice quadrata della distanza ( $n=0.5$ ), le onde di volume generate alla punta subiscano un decadimento proporzionale alla distanza ( $n=1$ ), mentre l'attenuazione maggiore si ha per le onde di volume che si propagano in superficie il cui decadimento è proporzionale al quadrato della distanza ( $n=2$ ). Considerando poi l'attenuazione per material damping si osserva che le vibrazioni a basse frequenze subiscono un decadimento minore di quelle alle alte frequenze, inoltre assume un ruolo fondamentale il terreno sia per la capacità di propagare le onde a maggiore (terreni densi o consistenti) o minore velocità (terreni sciolti o molli), quindi generare una minore o maggiore attenuazione, sia per la capacità, generalmente maggiore per i terreno granulare minore per il coesivo, di dissipare per attrito l'energia meccanica, in relazione al livello di deformazione cui è sottoposto al passaggio dell'onda.

L'impiego della (2), per la valutazione dell'attenuazione, è possibile solamente nel caso in cui il valore dell'ampiezza di vibrazione sia noto in una particolare posizione e quando sia possibile prevedere con certezza i tipi d'onda che si propagano; è quindi generalmente accettato l'impiego di relazioni cosiddette di pseudo-attenuazione di natura empirica, che vengono ritenute soddisfacenti (Woods, 1997) per la valutazione dell'attenuazione a distanza relativamente breve dalla sorgente. Tali relazioni descrivono l'attenuazione della vibrazione con la distanza, attraverso il valore della velocità di picco delle particelle di terreno, tale grandezza infatti è riconosciuta come la più adatta a caratterizzare con accuratezza il moto nel campo delle medie frequenze, per le quali in genere la strutture sono più sensibili, ed è quindi la grandezza di riferimento in diverse normative internazionali riguardanti i danni indotti dalle vibrazioni sugli edifici.

In fase previsionale il valore del modulo della velocità di picco delle particelle (peak particle velocity, PPV) o il valore assoluto della componente di picco della velocità (peak component particle velocity, PCPV), si ricorda che quest'ultimo in genere risulta fino al 25% inferiore a PPV (Athanasopoulos e Pelekis, 2000), possono essere valutati attraverso espressioni del tipo log-log lineari o quadratiche derivate dall'elaborazione statistica di dati provenienti da numerose misure sperimentali (Wiss J.F., 1981; Attewell and Farmer, 1973; Moore et al., 1995; Attewell et al., 1992):

$$\log(\text{PCPV o PPV}) = \log(c) + m \log(1/r) \quad (4.1)$$

$$\log(\text{PCPV o PPV}) = \log(c) + m \log(E_0^{0.5}/r) \quad (4.2)$$

$$\log(\text{PCPV o PPV}) = \log(c) + m \log(E_0^{0.5}/r) + n \log(E_0^{0.5}/r)^2 \quad (4.3)$$

con PCPV o PPV [mm/s],  $r$  [m] distanza dalla sorgente,  $E_0$  [J] energia fornita dal mezzo d'infissione ed  $c$ ,  $m$ ,  $n$  coefficienti dipendenti dal tipo di regressione utilizzata (tabella 3). Le (4) sono strettamente correlate alle caratteristiche geotecniche dei siti in cui sono stati registrati i dati sottoposti ad analisi statistica e quindi il loro utilizzo non può essere esteso a priori, senza una validazione attraverso l'analisi di dati raccolti localmente.

Tabella 3. Coefficienti di correlazione ottenuti per infissione tramite battipalo in vari siti del Regno Unito (Attewell et al., 1992).

Regressione	Curva	c	m	n
(4.2)	media	0.794	0.726	0
(4.2)	media+st.dev.	2.218	0.726	0
(4.3)	media	0.302	1.38	-0.234
(4.3)	media+st.dev.	0.845	1.38	-0.274

In tale ambito la BS 5228 - parte 4 suggerisce, per la valutazione di PPV, l'impiego della relazione (4.2) con  $m=1$  e  $c=0.75$ , mentre l'Eurocodice 3 - parte 5 suggerisce la stessa relazione con  $m=1$  e per il coefficiente  $c$  i valori riportati in tabella 4, in funzione del tipo di terreno e della modalità d'infissione.

Tabella 4. Eurocodice 3 - parte 5, tabella C.1.

Metodo d'infissione	Tipo di terreno	Coefficiente c
Impatto	Materiali coesivi molto compatti, materiali granulari densi, rocce, materiale di riporto con trovanti.	1.00
	Materiali coesivi compatti, materiali granulari mediamente densi, materiale di riporto compattato.	0.75
	Materiali coesivi molli, materiali granulari sciolti, terreni organici, materiale di riporto non compattato.	0.50
Vibrazione	Tutti i terreni	0.70

Studi sulla propagazione delle onde di volume e di superficie condotti con l'ausilio di modellazione numerica agli elementi finiti in ambito visco-elastico (Ramshaw et al., 1998) confermano i fenomeni di attenuazione sopra citati ed evidenziano come ad una certa distanza dal palo prevalgano le onde di superficie.

### 3 NORMATIVE RIGUARDANTI LE VIBRAZIONI E I LORO EFFETTI SUGLI EDIFICI

Le normative internazionali forniscono i livelli di accettabilità delle vibrazioni, in termini di velocità di picco delle particelle, con riferimento all'edilizia sia civile che industriale.

In figura 2 sono riportati i limiti di PCPV o di PPV, misurati sulle strutture, suggeriti dalle normative tedesche (DIN 4150 - PCPV), svizzere (SN 640312 - PPV) ed inglesi (BS 7385 - PCPV e 5228-4 PPV), in funzione delle frequenze dominanti del moto vibratorio.

Si osserva in genere un incremento del valore di

velocità accettabile all'aumentare della frequenza, in quanto nel campo delle alte frequenze il moto vibratorio risulta in genere meno dannoso per gli edifici. In particolare le norme tedesche e svizzere distinguono tra le diverse tipologie di fabbricato, prescrivendo valori limite diversi.

L'Eurocodice 3 fornisce dei valori limite di PPV (tabella 5) in base ai quali gli edifici dovrebbero essere tutelati dai danni strutturali e nel contempo dovrebbe diminuire la probabilità della comparsa di danni estetici. I valori riportati in tabella 5 devono essere ridotti del 50% in presenza di edifici già danneggiati o dove si prevedano frequenze al di sotto di 10 Hz.

In tale ambito la norma UNI 9916-2004 richiama le norme tedesche (DIN 4150), inglesi (BS 7385 e 5228-4) e svizzere (SN 640312).

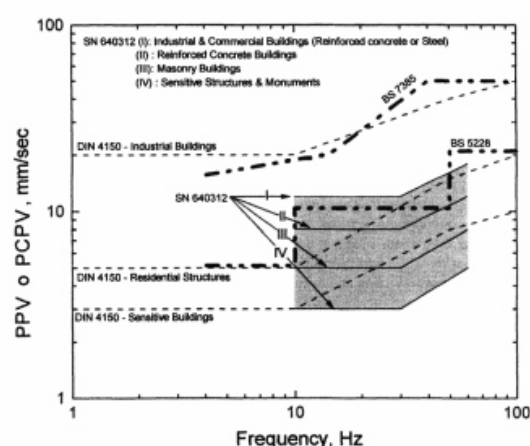


Figura 2. Normative internazionali - valore limite di PPV o PCPV per diverse tipologie di fabbricato (adattato da Athanasopoulos G.A. e Pelekis P.C., 2000).

Tabella 5. Eurocodice 3 - parte 5, tabella C.3.

Tipologia di fabbricato	PPV (mm/s)	
	Vibraz. permanenti	Vibraz. transitorie
Beni archeologici, monumenti	2	4
Edifici residenziali	5	10
Edifici commerciali leggeri	10	20
Edifici industriali pesanti	15	30
Servizi interrati	25	40

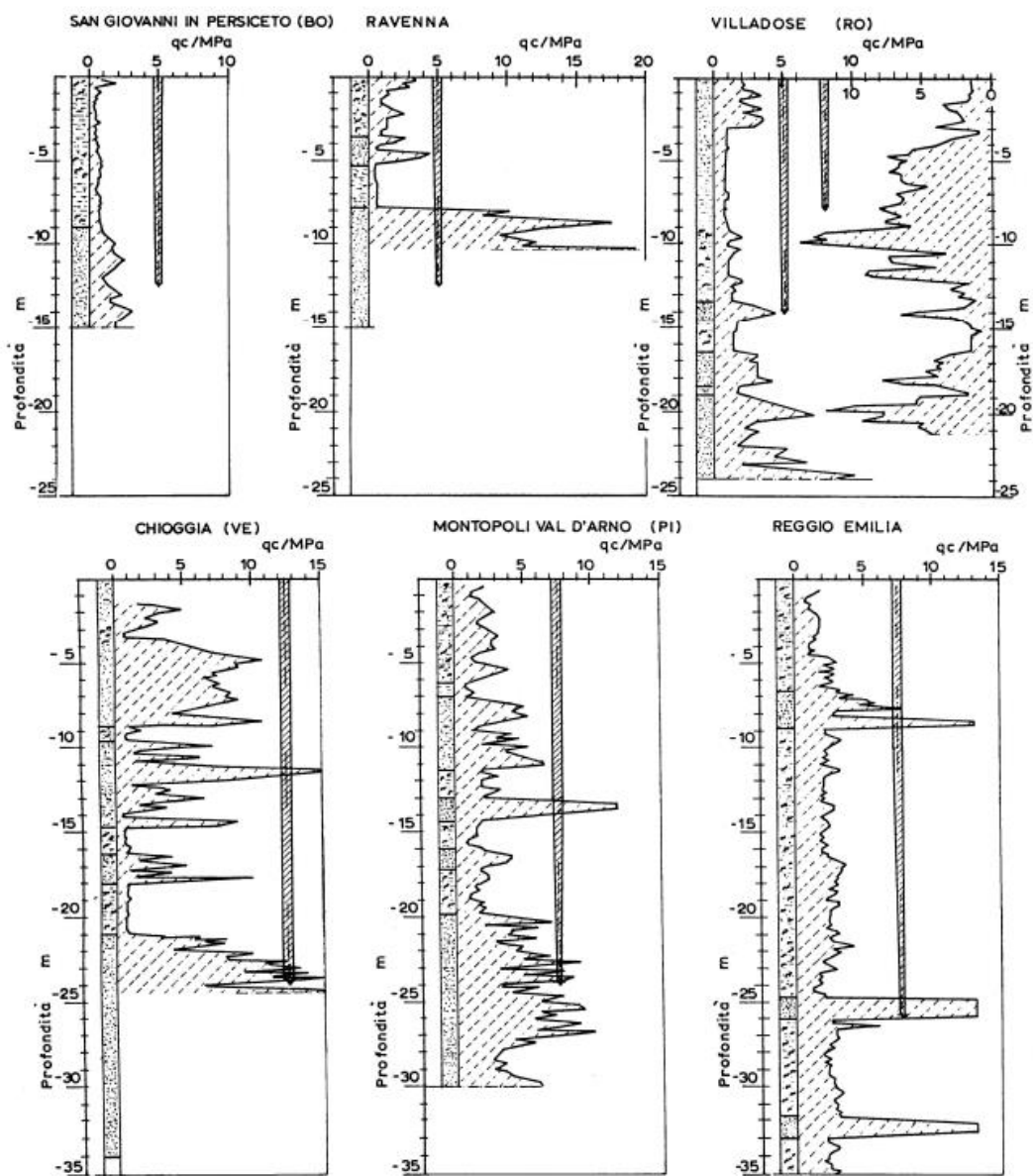


Figura 3. Caratteristiche stratigrafiche rappresentative dei siti studiati.

#### 4 VIBRAZIONI INDOTTE DALL'INFISSIONE IN TERRENI ALLUVIONALI

Sono stati analizzati i dati provenienti da una serie di rilievi vibrometrici condotti durante l'infissione di pali prefabbricati in terreni di natura alluvionale.

La figura 3 mostra alcune stratigrafie rappresentative dei siti interessati dallo studio; si tratta di terreni costituiti da formazioni di natura coesiva (limi argillosi e argille limose) da poco consistenti a consistenti alternati a formazioni granulari da mediamente dense a dense, con la superficie libera della falda che si colloca a circa 1-2 m da piano campagna.

I pali prefabbricati in calcestruzzo armato centrifugato, cilindrici e troncoconici, hanno lunghezze variabili tra 8 m e 28 m (giuntati) e diametri alla punta variabili tra 0.24 m e 0.50 m.

L'infissione è stata eseguita con battipali diesel tipo Delmag con energie nominali variabili da un massimo di 145.37 kJ (Delmag D46) ad un minimo di 53.23 kJ (Delmag 16-32) ed con battipalo tipo Banut 400 con energia nominale pari a 27.46 kJ. Si tratta di attrezzature di normale impiego per l'infissione di pali in tali terreni, fino a profondità dell'ordine di 30 m.

Il monitoraggio ha previsto l'impiego di geofoni 3D e accelerometri, installati, direttamente sulle strutture esistenti o su picchetti, posti a distanze variabili tra 8 m e 185 m dal punto d'infissione. Sono state rilevate le velocità sul piano verticale (vert) e sul piano orizzontale, in direzione radiale (rad) e tangenziale (tang) rispetto al punto d'infissione.

Le osservazioni a disposizione sono state elaborate, con riferimento ai criteri di previsione in termini di PCPV definiti dalle (4), ottenendo, per i terreni in oggetto, le seguenti espressioni:

##### MISURE SU TERRENO

media

$$\log(\text{PCPV}) = \log(0.163) + 1.1329 \log(E_0^{0.5}/r) \quad (5.1)$$

media + st.dev.

$$\log(\text{PCPV}) = \log(0.442) + 1.1329 \log(E_0^{0.5}/r)$$

##### MISURE SU FONDAZIONE

media

$$\log(\text{PCPV}) = \log(0.308) + 0.6412 \log(E_0^{0.5}/r) \quad (5.2)$$

media + st.dev.

$$\log(\text{PCPV}) = \log(0.764) + 0.6412 \log(E_0^{0.5}/r)$$

In figura 4 si riporta traccia delle (5) con riferimento alla distribuzione dei dati sperimentali ed in figura 5 si riportano le curve di previsione (5) media+st.dev. per bassi, medi e alti valori dell'energia fornita dal battipalo.

E' interessante evidenziare come l'entità delle vibrazioni sul terreno risultino circa il doppio di quelle osservate nelle strutture, in relazione alla differenza delle masse interessate dal moto vibratorio.

Il grafico di figura 6 mostra i valori di frequenza registrati durante il processo d'infissione ed i relativi valori di velocità, distinti in base alla distanza dalla sorgente di vibrazione. E' possibile notare come i valori rilevati nei siti caratterizzati da alternanze di strati coesivi e granulari rientrino generalmente entro i limiti suggeriti dalle norme DIN 4150 e dall'Eurocodice 3 per gli edifici

residenziali, quando si impiegano i battipali con l'accortezza di regolare l'energia in relazione alla distanza dei fabbricati.

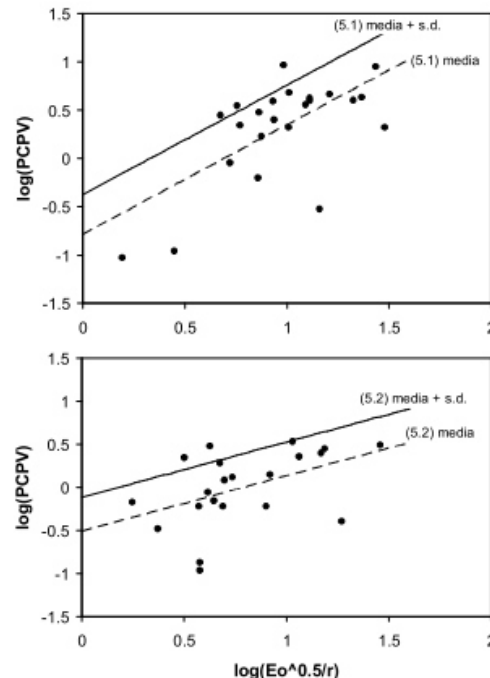


Figura 4. Distribuzione dei dati rilevati rispetto alle rette di regressione (5).

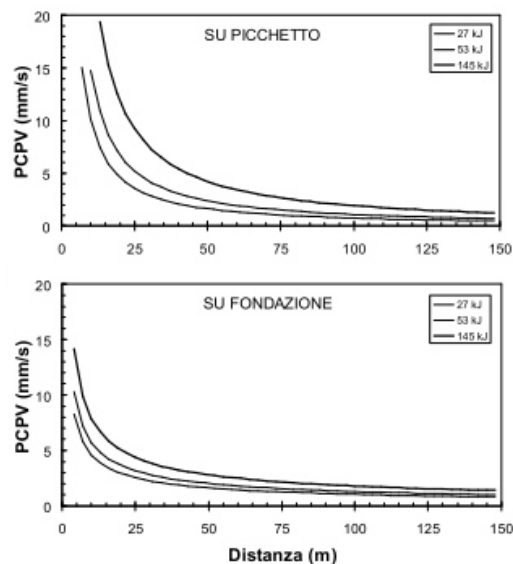


Figura 5. Curve di previsione della PCPV con la distanza, al variare dell'energia fornita.



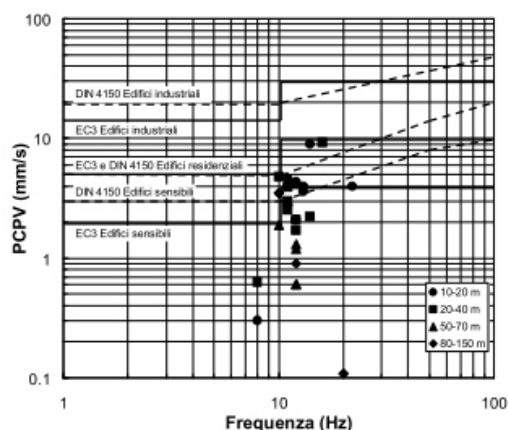


Figura 6. Confronto tra i dati osservati e i limiti di velocità suggeriti dalle norme DIN ed EC3.

## 5 CONCLUSIONI

Nell'articolo si sono esaminati criticamente i dati ottenuti da misure di vibrazioni indotte, sul terreno e sugli edifici, dall'infissione di pali prefabbricati in formazioni alluvionali, costituite da alternanze di strati argillosi, di consistenza da media a compatta, e sabbie di densità da media ad elevata.

Si tratta di terreni in cui i pali infissi hanno sempre trovato larga diffusione, nell'edilizia residenziale ed industriale, che però nel tempo è diminuita per effetto della crescente urbanizzazione con la conseguente riduzione delle distanze tra gli edifici.

Le costruzioni adiacenti ai cantieri costituiscono oggetto di attenzione non solo per i possibili danni sulle opere, generalmente di carattere estetico, ma anche per il disturbo nei confronti delle persone.

I fenomeni di danno collegati alle operazioni di infissione dipendono sia dai valori di frequenza che dall'energia impiegata. E' noto infatti che l'impiego di attrezzature che operano nel campo delle alte frequenze consente di minimizzare i fenomeni sopracitati.

Si deve però considerare che le alte frequenze producono sui pali in calcestruzzo, ad armatura lenta, estese fessurazioni, per questo motivo si utilizzano attrezzature che operano nel campo delle basse frequenze.

In tal caso il livello di vibrazione e di disturbo, possono essere ridotti solamente attraverso la regolazione dell'energia impiegata per l'infissione, come evidenziato dai risultati presentati nell'articolo. Tale operazione non è sempre possibile con gli usuali battipali diesel in quanto con energie al di sotto di certi valori le operazioni di infissione risultano difficoltose, con avanzamenti molto contenuti e notevole disturbo nei confronti delle persone, d'altro canto l'impiego di un battipalo ad elevata energia, sebbene fornisca un avanzamento elevato, nel contempo può causare danni conseguenti alle vibrazioni prodotte.

Per questi motivi quindi assume particolare rilevanza la possibilità di disporre di attrezzature in grado di

regolare l'energia in modo più efficiente, come i classici battipali a caduta libera ed idraulici.

Inoltre, in situazioni particolarmente delicate, risulta opportuno prevedere anche l'impiego di prefiori o iniezioni con acqua in pressione.

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## ABSTRACT

### GROUND VIBRATIONS FROM PILE DRIVING

Keywords: driven piles, vibrations, damage, National and European standards

In the urban environment, decisions regarding the planning of foundation works are often influenced by the effects which construction technologies may have on nearby buildings.

In particular, driving piles and sheetpiles induces vibrations which propagate in the soil, often generating disturbance and sometimes damage to particularly vulnerable structures.

This paper reports and critically analyses the results of vibration measurements carried out on sites in alluvial soil, during driving of cylindrical and tapered prefabricated piles, using equipment applying various degrees of energy.

Statistical processing of data, with reference to the studied soils, allowed us to calibrate pseudo-attenuation relations in order to assess vibratory motion in terms of peak particle velocity, with increasing distance from the source and varying the energy normally used for pile-driving.

Measured velocity values are compared with the threshold limits recommended by UNI, DIN and EC3 European norms, and suggestions are made as to the criteria on which to base choices of pile-driving equipment, with reference to some structural types characteristic of the urban environment.



# Structural vibration

## Part 3: Effects of vibration on structures

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### Foreword

This standard has been prepared by Technical Committee *Schwingungsfragen im Bauwesen; Einwirkungen auf bauliche Anlagen* of the *Normenausschuß Bauwesen* (Building and Civil Engineering Standards Committee).

### Amendments

The following changes have been made to the May 1986 edition.

- The standard now also covers the effects of vibration on buried pipework.
- The standard has been revised in form and content to reflect the current state of the art.

### Previous editions

DIN 4150-3: 1975-09, 1986-05.



Continued on pages 2 to 11.

Translation by DIN-Sprachendienst.

In case of doubt, the German-language original should be consulted as the authoritative text.

## 1 Scope

This standard specifies a method of measuring and evaluating the effects of vibration on structures designed primarily for static loading. It applies to structures which do not need to be designed to specific standards or codes of practice as regards dynamic loading.

This standard gives guideline values which, when complied with, will not result in damage that will have an adverse effect on the structure's serviceability. In some cases, guideline values for a simplified evaluation are also given.

## 2 Normative references

This standard incorporates, by dated or undated reference, provisions from other publications. These normative references are cited at the appropriate places in the text, and the titles of the publications are listed below. For dated references, subsequent amendments to or revisions of any of these publications apply to this standard only when incorporated in it by amendment or revision. For undated references, the latest edition of the publication referred to applies.

- DIN 1311-1 Vibration and shock – Vibration systems – Concepts, classification\*)
- DIN V 4150-1 Structural vibration – Principles and measurement of vibration parameters\*\*)
- DIN 4150-1 Structural vibration – Preliminary measurement of vibration parameters\*)
- DIN 45669-1 Mechanical vibration and shock measurement – Measuring equipment
- DIN 45669-2 Mechanical vibration and shock measurement – Measurement procedure
- DIN EN 1594 Gas supply systems – Pipelines – Maximum operating pressure over 16 bar – Functional requirements\*)

## 3 Concepts

For the purposes of this standard, the following definitions apply in addition to those defined in DIN 1311-1.

### 3.1 Vibration

Mechanical vibration of solid bodies which may cause damage or discomfort.

### 3.2 Damage

Any permanent effect of vibration that reduces the serviceability of a structure or one of its components.

### 3.3 Guideline value

A value obtained through experience; compliance with this value ensures that damage will not occur.

### 3.4 Short-term vibration

Vibration which does not occur often enough to cause structural fatigue and which does not produce resonance in the structure being evaluated.

### 3.5 Long-term vibration

All types of vibration not covered by the definition of 'short-term vibration' in subclause 3.4.

## 4 Principles of evaluating the effects of vibration on structures

### 4.1 General

Clauses 5 and 6 specify methods of measuring and evaluating vibration parameters. If these methods are not used, then the dynamic stresses occurring in the structure are to be determined by measurement or analysis (e.g. as in subclauses 4.2 and 4.3, respectively) and the results then compared with the permissible stresses, taking their frequency of occurrence into account. Note that the methods described in subclauses 4.2 and 4.3 are not suitable for assessing minor damage as defined in subclause 4.5.

Sometimes, vibration cannot be classified as being only short-term or only long-term as defined in subclauses 3.4 and 3.5, respectively. In such cases, it shall be evaluated on the basis of both clause 5 and clause 6.

\*) Currently at draft stage.

\*\*) 1975 edition.

Where necessary, foundation displacement as an indirect consequence of vibration shall also be taken into consideration (cf. Appendix C).

## 4.2 Determining stresses by measurement

By measuring the strain in a vibrating building component and applying the mass law, the stresses present can be inferred.

The amplitude and frequency of the measured vibration displacement, velocity or acceleration can be used in stress/strain calculations.

The stresses in beams and slabs vibrating close to resonance can be approximated on the basis of the vibration velocity amplitude, provided the measurement is made at the point of the greatest amplitude. In this case, the boundary conditions and stiffness of the component need not be known (cf. subclause 6.2).

## 4.3 Determining stresses by analysis

The analysis of stresses shall be performed using state-of-the-art methods. Values used in the analysis may be obtained by means of the predictive method described in DIN V 4150-1 or DIN 4150-1.

## 4.4 Permissible stresses

Verification of stability shall be carried out using the safety factors specified in the relevant standards and regulations for additional dynamic loading, taking into account the type and duration of the dynamic loads imposed, the measurement method, the characteristics of the building materials and the type of construction. If necessary, fatigue strength shall also be verified. Stress-number curves may be used to establish, as a function of the number of expected stress reversals, the stress limits, stress amplitudes, limits of strain and similar parameters for the building materials, building components and junctions.

A detailed analysis of fatigue strength may be dispensed with if, for the stability analysis, the dynamic load components are multiplied by a factor of 3.

Fatigue analysis is not required if the dynamic load component is less than 10 % of the permissible static stress.

## 4.5 Evaluating serviceability

Examples of a reduction in the serviceability of a building or building component due to the effects of vibration include:

- the impairment of the stability of the building and its components;
- a reduction in the bearing capacity of floors.

For structures as in lines 2 and 3 of table 1, the serviceability is considered to have been reduced if

- cracks form in plastered surfaces of walls;
- existing cracks in the building are enlarged;
- partitions become detached from loadbearing walls or floors.

These effects are deemed 'minor damage'.

## 4.6 Effects of vibration on soil

Strong vibration can cause settlement of soil, primarily in the case of loose to medium-dense, non-cohesive soil such as sand and gravel; this can also lead to foundation settlement, especially where there is frequent vibration or uniformly graded sand or soil beneath the groundwater level. For more information, see Appendix C.

# 5 Evaluating effects of short-term vibration

## 5.1 Effects on the structure as a whole

Numerous measurements of vibration velocity in building foundations have provided empirical values which give guidance on the evaluation of short-term structural vibration. Evaluations as in this standard are based on the maximum absolute value of the velocity signals,  $|v|_{i,max}$ , for the three components (where  $i = x, y$  or  $z$ ) of the unweighted velocity signals,  $v_i(t)$ , measured on the building foundation (this parameter is referred to below as  $v_i$  for short). See subclause 5.4 for details of measurement.

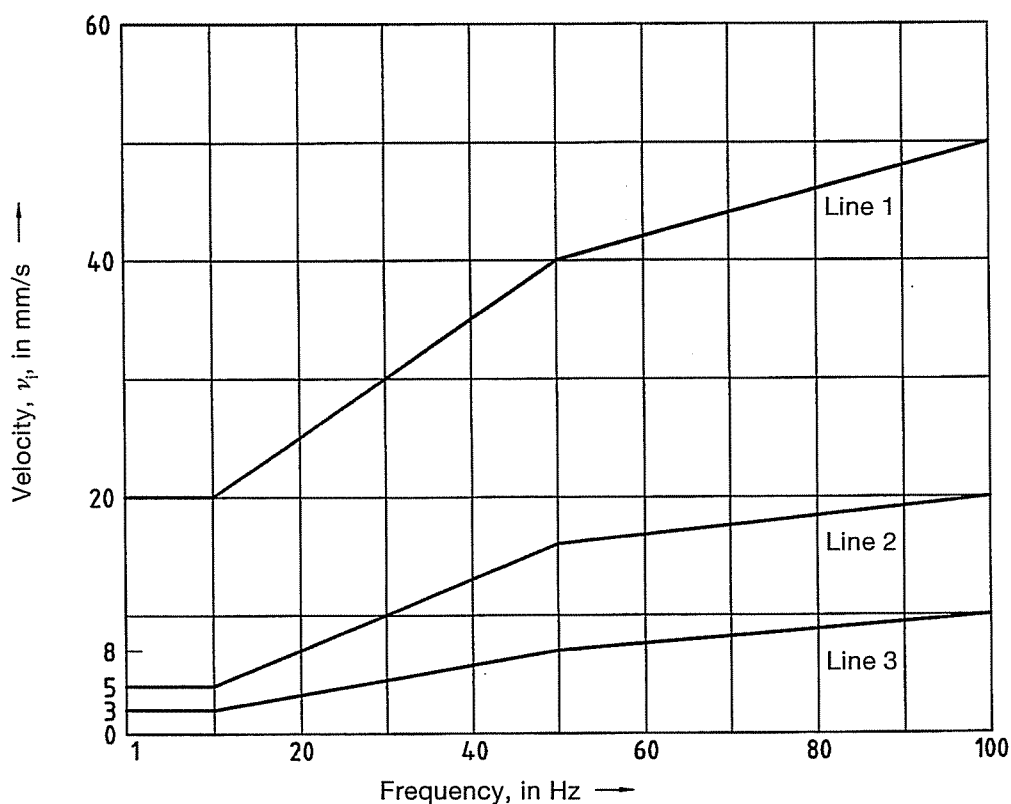
The vibration measured in the plane of the highest floor resting on external walls also provides significant information for this evaluation, taking the maxima of the two horizontal components as a basis. Measurements taken at this point in accordance with subclause 5.4 may be used to determine the horizontal response of the structure to the excitation at the foundation.

Table 1 and figure 1 give guideline values for  $v_i$  at the foundation and in the plane of the highest floor of various types of building. Experience has shown that if these values are complied with, damage that reduces the serviceability of the building will not occur. If damage nevertheless occurs, it is to be assumed that other causes are responsible. Exceeding the values in table 1 does not necessarily lead to damage; should they be significantly exceeded, however, further investigations are necessary.

**Table 1: Guideline values for vibration velocity to be used when evaluating the effects of short-term vibration on structures**

Line	Type of structure	Guideline values for velocity, $v_i$ , in mm/s			
		Vibration at the foundation at a frequency of			Vibration at horizontal plane of highest floor at all frequencies
		1 Hz to 10 Hz	10 Hz to 50 Hz	50 Hz to 100 Hz*)	
1	Buildings used for commercial purposes, industrial buildings, and buildings of similar design	20	20 to 40	40 to 50	40
2	Dwellings and buildings of similar design and/or occupancy	5	5 to 15	15 to 20	15
3	Structures that, because of their particular sensitivity to vibration, cannot be classified under lines 1 and 2 and are of great intrinsic value (e.g. listed buildings under preservation order)	3	3 to 8	8 to 10	8

\*) At frequencies above 100 Hz, the values given in this column may be used as minimum values.



**Figure 1: Curves for guideline values specified in table 1 for velocities measured at the foundation**

To determine which frequency ranges shown in table 1 apply, take the frequency which occurs within the relevant velocity range, special care being necessary in the measurement of low frequencies. For analytical purposes, the character of the signal shall also be taken into consideration, for instance by means of suitable data windows (cf. Appendix D).

For civil engineering structures (e.g. reinforced concrete constructions used as abutments or foundation pads), the values in line 1 of table 1 may be increased by as much as a factor of two, provided no hazards arise as a result of mechanical processes in the ground.

## 5.2 Effects on floors

Where short-term vibration causes floors to vibrate, if  $v_z$  is no greater than 20 mm/s when measured at the point of maximum velocity (which is usually at the centre of the floor), a reduction in the serviceability of the floor is not to be expected. In the case of buildings as in line 3 of table 1, it may be necessary to lower this value to prevent minor damage.

## 5.3 Effects on buried pipework

Table 2 gives guideline values for evaluating the effects of vibration on buried pipework. It is assumed that the pipes have been manufactured and laid using current technology; if this is not the case, special considerations will have to be made. Additional considerations need also be made where mechanical processes in the ground could have deleterious effects on pipes, or where there are different stress conditions at junctions (e.g. junctions with the structure).

The values given in table 1 for foundations also apply to the first two metres (nearest the building) of gas and water service pipes. For information regarding gas supply pipelines, see DIN EN 1594.

Drain pipes shall be evaluated using the values given in line 3 of table 2.

**Table 2: Guideline values for vibration velocity to be used when evaluating the effects of short-term vibration on buried pipework**

Line	Pipe material	Guideline values for velocity measured on the pipe, $v_i$ , in mm/s
1	Steel (including welded pipes)	100
2	Clay, concrete, reinforced concrete, pre-stressed concrete, metal (with or without flange)	80
3	Masonry, plastic	50

## 5.4 Measurement

Instruments used to perform measurements as in this standard shall meet the requirements specified in DIN 45669-1, and the procedure shall be as in DIN 45669-2. To measure vibration in foundations, the pick-ups for the three directions of measurement shall be placed close together on the ground floor of the building to be investigated, either at the foundation of the outer wall, on the outer wall itself, or in a recess in that wall. In buildings without a basement, the point of measurement shall be no more than 0,5 m above the ground. Measurement points shall preferably be on the side of the structure that faces the source of excitation. The time history of the vertical vibration (z-axis) and horizontal vibration (x- and y-axes, at right angles to each other) shall be recorded, with one of the directions of measurement running parallel to a side wall of the building. For structures with a large ground floor area, simultaneous measurements shall be made at several locations.

In addition to the measurements made on the foundation and the highest floor, a measurement in the vertical direction may also have to be made on the floors on which the strongest vibration is expected; in this case, the point of measurement should be in the centre of the floor (cf. subclause 5.2).

Pick-ups for measurements in the highest floor shall be placed on or immediately next to structural masonry so that the two horizontal directions of measurement, x and y, are at right angles to each other, with one direction running parallel to a side wall.

When carrying out measurements on pipework, pick-ups shall be placed directly on the pipes whenever possible. As an alternative, the pick-up may be placed on the ground surface directly above the pipe, although in this case, it is only possible to make estimates (see Appendix D.1).

A test report as in Appendix A shall be drawn up for each measurement.

## 6 Evaluating effects of long-term vibration

### 6.1 Effects on the structure as a whole

Table 3 gives guideline values for the highest value of the two horizontal components measured in the top floor, for different types of building. Experience has shown that if these values are complied with, damage will not occur. Exceeding the values in table 3 slightly does not necessarily lead to damage. Should they be considerably exceeded, the stresses may be determined as described in subclauses 4.2 and 4.3 and evaluated as in

subclause 4.4. In the case of multi-storey frame structures, the dynamic stress component can also be determined from the relative displacement of the ends of the vertical members.

If a building is subjected to harmonic vibration, then the maximum values can also occur in floors other than the top floor, or in the foundation. The values given in table 3 also apply in these cases.

When other points of reference are used, separate analysis is required.

**Table 3: Guideline values for vibration velocity to be used when evaluating the effects of long-term vibration on structures**

Line	Type of structure	Guideline values for velocity, $v_1$ , in mm/s, of vibration in horizontal plane of highest floor, at all frequencies
1	Buildings used for commercial purposes, industrial buildings, and buildings of similar design	10
2	Dwellings and buildings of similar design and/or occupancy	5
3	Structures that, because of their particular sensitivity to vibration, cannot be classified under lines 1 and 2 and are of great intrinsic value (e.g. listed buildings under preservation order)	2,5

## 6.2 Effects on floors

To evaluate vibration in components such as floors and walls, the dynamic loading may be determined as in subclauses 4.2 and 4.3.

In the case of flexural vibration close to resonance, which often occurs when floors vibrate at high magnitudes, the additional dynamic stress can be approximated using the method mentioned in subclause 4.2 as described below.

For beams and one-way spanning solid slabs of rectangular cross section (i.e.  $y_{\max}/i = 1,73$ , where  $y_{\max}$  is the outer fibre distance and  $i$  is the radius of inertia) with a constant stiffness and weight loading, and for vibration with a natural mode, the maximum bending stress,  $\hat{\sigma}_{\max}$ , is defined by equation (1), regardless of the dimensions of the vibrating system:

$$\hat{\sigma}_{\max} = 1,73 (E_{\text{dyn}} \rho G_{\text{tot}}/G_{\text{beam}})^{0,5} k_n \hat{v}_{\max} \quad (1)$$

where

- $\hat{v}_{\max}$  is the peak velocity along the beam length;
- $E_{\text{dyn}}$  is the dynamic modulus of elasticity of the material;
- $\rho$  is the material density;
- $G_{\text{tot}}/G_{\text{beam}}$  is the coefficient of loading, where the beam is to accommodate evenly distributed loads in addition to its self-weight;
- $G_{\text{tot}}$  is the self-weight of the beam, plus other loads;
- $k_n$  is the eigenmode coefficient.

The eigenmode coefficient is dependent on the boundary conditions and the degree of the mode. Both of these have only a slight influence; however, in practice, the value for  $k_n$  lies between 1 and 1,3. For two-way spanning slabs, the bending stress so calculated is also to be considered a maximum.

Experience has shown that vertical vibration velocities up to 10 mm/s do not cause damage in floors of structures as in lines 1 and 2 of table 3, even if the maximum design stresses are fully utilized. Such vibration is very clearly perceptible. For structures as in line 3 of table 3, no guideline value can be given for vertical vibration.

Minor damage (cf. subclause 4.5) should not be automatically attributed to dynamic loading and further investigations are necessary.

## 6.3 Effects on buried pipework

The guideline values given in table 2 may be reduced by 50 % without further analysis when evaluating the effects of long-term vibration on buried pipework.

The restrictions given in subclause 5.3 apply here by analogy.

## 6.4 Measurement

If a building is subjected to harmonic vibration, measurements shall be taken on several floors simultaneously in order to correctly determine the vibrational mode. For vibration having the lowest natural mode, it is normally



sufficient to take measurements on the top floor. The lowest natural frequency of horizontal vibration in buildings with about five or more storeys,  $f_i$ , in Hz, may be taken to be approximately  $10/n$  (where  $n$  is the number of storeys).

When evaluating horizontal vibration in the structure as a whole, it may be necessary in special cases to take into account possible rotational movements in the floor plane and any rigid rotation.

The natural frequency of floors is normally greater than 10 Hz, and in most cases, only vertical movements are significant. The vertical vibration shall thus be measured at the point of maximum velocity, which is usually at the centre of the floor.

A test report as in Appendix A shall be drawn up for each measurement.

## Appendix A

### Sample test report form

The test report shall include the information listed below.

**Table A.1: Test report form**

Line	Type of information	Details
1	General: a) Testing agency b) Client c) Contract identification d) Person carrying out measurement e) Time and date of measurement	
2	Type of vibration: a) Source b) Operating conditions	Blasting (charge, ignition stages, number of drill holes, series, etc.) Pile driving (equipment used, type of pile used) Machinery (speed, load, etc.) Traffic (rail traffic, trucks, etc.) Frequency of occurrence
3	Structure: a) Designation b) Classification c) Description	Address Type of building according to the tables in this standard Type of structure, size, foundation, structural condition
4	Site and location a) of source of vibration b) of the measurement points and their distance from the source, and measurement direction	Sketches giving heights
5	Environmental conditions	Details of rock and soil, ground water, structural condition of building, weather conditions (frost, storm, etc.), extraneous sources of vibration (e.g. traffic)
6	Subjective observations	Perceptible secondary effects (e.g. rattling of objects)
7	Measuring chain: a) Pick-ups, natural frequency of equipment, damping coefficient, frequency response, operating frequency range b) Signal conditioning equipment c) Recording devices d) Tools for analysis	Accelerators, velocity or displacement pick-ups  Filters, amplifiers Magnetic tape recorder, plotters, PCs Frequency analyzers, software
8	Results of measurement: a) Measured quantities and frequencies b) Derived quantities c) Duration and occurrence of effects	
9	Signatures	

## Appendix B

### Measures for limiting the effects of vibration

Normally, vibration is transmitted through the ground and decays with increasing distance from its source. For this reason, the effects of vibration can be reduced by increasing the distance between the vibration source and receiver. (Airborne vibration plays a role only under special circumstances.)

The following measures may be used to limit the effects of vibration.

#### B.1 Measures taken at the vibration source

##### B.1.1 Measures against stationary vibration with harmonics, generated by machinery (e.g. oscillating screens, motors, compressors, sawmills)

- a) Balance machines.
- b) Provide or improve balancing systems.
- c) Change the speed, where resonance occurs.
- d) Isolate against vibration by placing the installation on an elastic element (for excitation at frequencies over 3 Hz).

##### B.1.2 Measures against shocks generated by machinery (e.g. forge hammers, presses, mills)

Isolate the installation against vibration.

##### B.1.3 Measures against vibration generated by traffic

- a) Build and maintain smooth road surfaces.
- b) Regularly maintain tracks of railways.
- c) Regularly maintain running gear of rail vehicles.
- d) Isolate railways against vibration.
- e) Reduce speed.

##### B.1.4 Measures against vibration generated by blasting

Modify the blasting technique (e.g. a different charge for each ignition stage, different firing orders or hole depths).

##### B.1.5 Measures against vibration generated by construction work

- a) Switch to low-vibration techniques.
- b) Use vibration hammers having higher vibration frequencies.
- c) Avoid resonance.

#### B.2 Measures taken at the vibration receiver (structure)

- a) Fit the structure with dynamic vibration absorbers (especially effective against resonance and where there is minimal damping in the structure).
- b) Isolate the structure against vibration (for excitation frequencies above 5 Hz).
- c) Adapt the structure to avoid resonance.

#### B.3 Measures taken along the transmission path

- a) Increase the distance between the source and the receiver (structure).
- b) In special cases, dig trenches or fit elements in the ground near the vibration source or the structure.

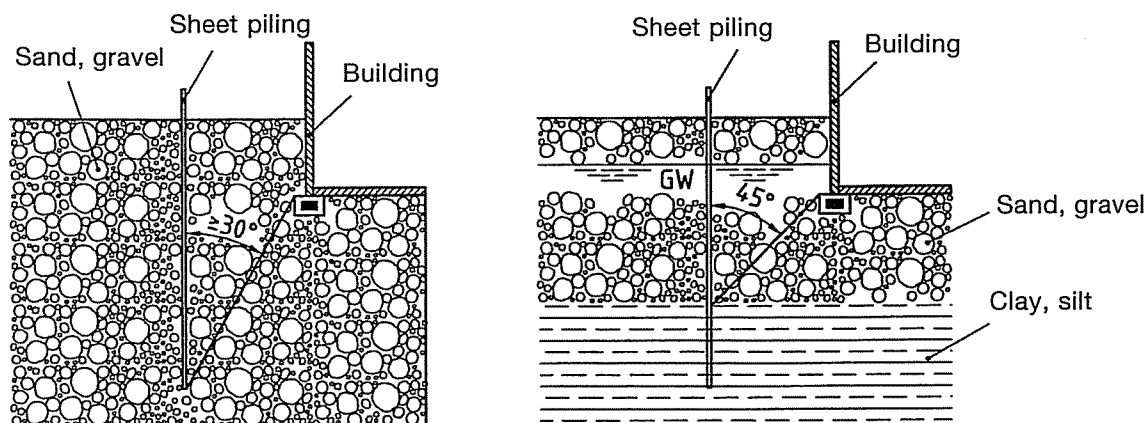
Where the foundation may be susceptible to differential settlement, adequate measures should be taken to strengthen the foundation accordingly (e.g. by sinking a deep foundation).

## Appendix C

### Effects of vibration on soil

Non-cohesive soil tends to settle, for instance when vibrating rams are used nearby to drive sheet piling. For this reason, the distance between the vibration source and the building foundation should be such that an angle of at least  $30^\circ$  to the vertical is formed as shown in figure C.1. For piling extending below the groundwater table, an angle of  $45^\circ$  is more suitable, as shown in the figure.

This tendency is considerably lower in the case of percussive driving methods (e.g. when using diesel or pneumatic rams).



GW – Groundwater table

Figure C.1: Distance between sheet piling and building (schematic)

Even at great distances from the vibration source, vibration-induced foundation settlement can still occur at vibration severities which are normally not expected to cause structural damage. For this to occur, the soil has to be very sensitive to vibration (as is non-cohesive, uniformly graded sand or silt, for instance), and the vibration has to be continuous or frequent.

Since few investigations have been made regarding dynamically-induced settlement, it is recommended that expert advice be sought.

Another effect vibration has on soil is liquefaction, when sand or silt at the groundwater level suddenly loses its bearing capacity as a result of dynamic effects. During earthquakes, this process can lead to damage as serious as the collapse of buildings. Since the vibration covered by this standard normally lies well under the vibration magnitudes which occur during strong earthquakes, these effects should only be expected under the most unfavourable circumstances.

## Appendix D

### Additional information on measurements on pipework and evaluation of frequencies

#### D.1 Vibration measurements on pipework

Measurements carried out to evaluate the effects of vibration on pipework should preferably be performed directly on the pipes. Wherever possible, buried pipes should be exposed only at the point of measurement. The pick-up should be mounted as described in subclause 5.3 of DIN 45669-2. The time history of the vibration should be measured in the  $z$ ,  $x$  and  $y$  directions, one of which should run along the pipe axis.

Any insulation at the point of measurement should be removed, although thin coatings have little effect on results. To provide the pick-up with a flat support surface, a concrete or plaster base may be mounted on the pipe.

Often, mounting pick-ups directly on the pipe can be quite involved. Where the vibration source is not immediately next to the pipework, or is nearby but much deeper than the pipes, measurements can be made on the ground surface. Previous investigations have shown that vibration measured on the surface is usually greater than that measured directly on pipes.

#### D.2 Role of frequency in evaluations

Table 1 gives guideline values for vibration at foundations as a function of frequency. It is assumed the following procedures will be carried out:

- 1) Finding the maximum velocity values over the time,  $v_i(t)$ .
- 2) Determining the significant frequencies,  $f_i$ , over  $v_i(t)$ .
- 3) Comparing the maximum velocities,  $v_i$ , with the values given in table 1 for this significant frequency.

NOTE: Narrow-band spectra are particularly suited for determining frequencies  $f_i$ . To reduce distortions of the spectra caused by the duration and form of the data window, the location and length of the latter have to be fitted to the time history,  $v_i(t)$ . Frequency weighting is not necessary.

EXAMPLE: When a construction machine is started up, short-term vibration occurs. The vibration components,  $v_x(t)$ ,  $v_y(t)$ , and  $v_z(t)$ , measured in the foundation of a nearby building have qualitatively similar time histories, as have the spectra. The maximum value of the vertical component,  $v_z$ , is considerably greater than those for  $v_x$  and  $v_y$ ; the horizontal components are therefore disregarded. Figure D.1 shows the time history of the vertical component  $v_z(t)$  with a maximum value of 5,1 mm/s.

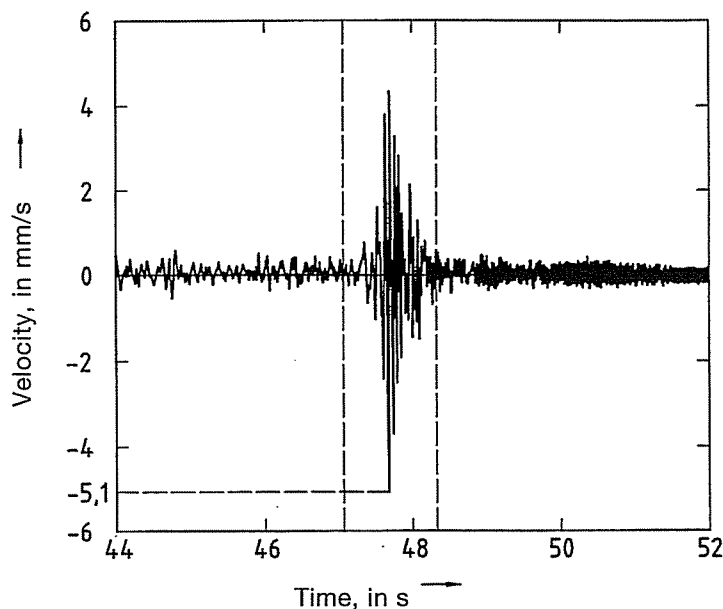


Figure D.1: Time history of the vertical vibration component, with a maximum of 5,1 mm/s

The main section of the vibration signal is enclosed by dashed lines and is enlarged in figure D.2.

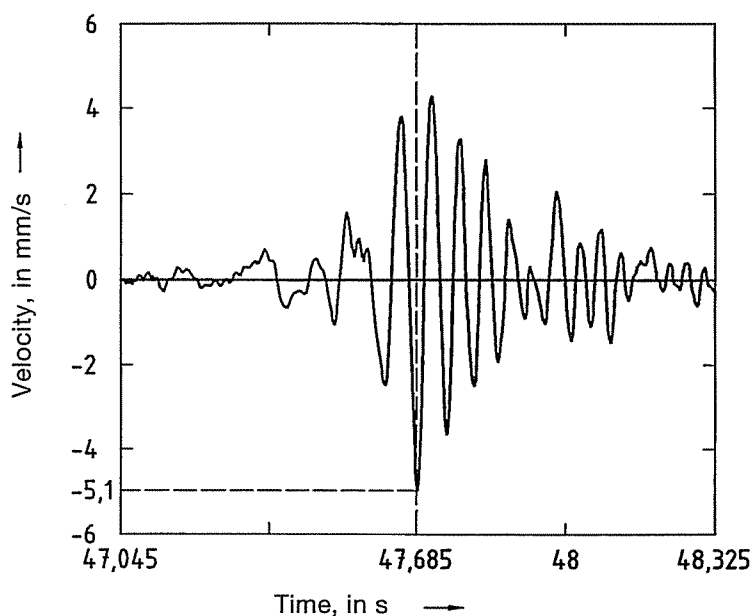
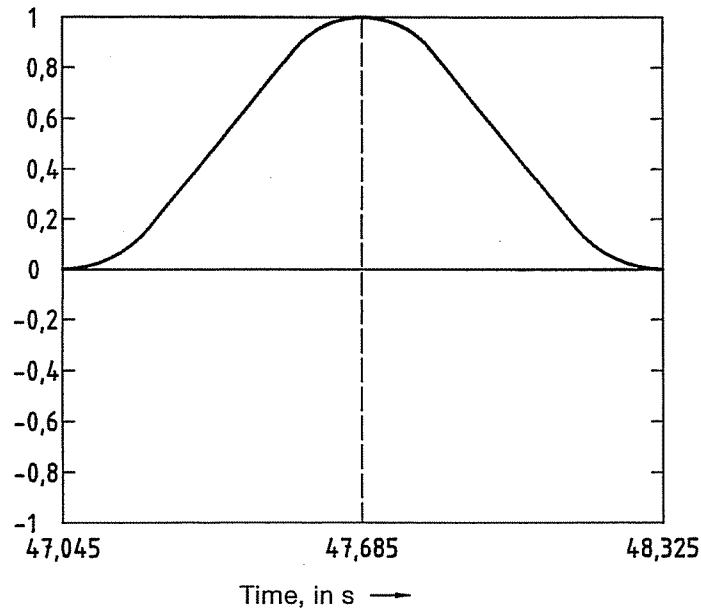


Figure D.2: Enlargement of 1,28 s section of time history shown in figure D.1

Before being transformed into a frequency range, the time history illustrated in figure D.2 is multiplied by the shifted Hanning window shown in figure D.3, given by

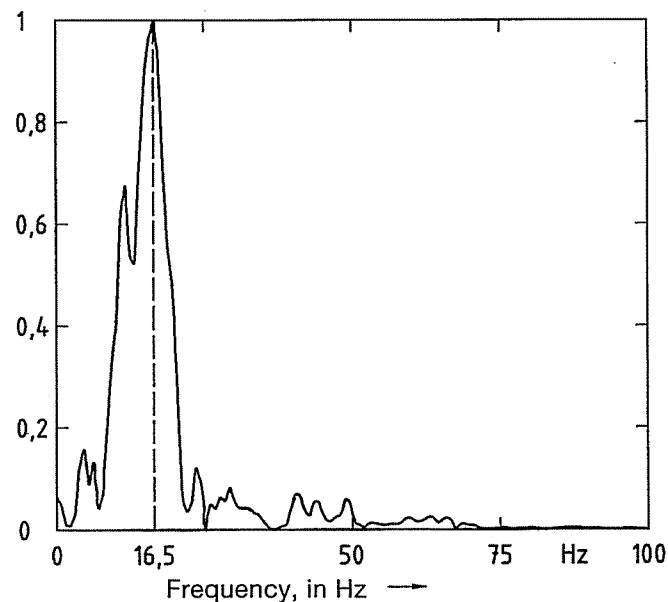
$$h_w(t) = \begin{cases} (1 - \cos(2\pi(t - t_0)/T_0))/2 & \text{for } t_0 \leq t \leq T_0 + t_0 \\ 0 & \text{otherwise} \end{cases}$$

The peak of the Hanning window corresponds to the maximum of  $v_z$ ; the length of the window has been adjusted to the length of the enlargement in figure D.2 (1,28 s).



**Figure D.3: Hanning window,  $(h_w(t))$ , fitted to  $v_z(t)$  (with  $t_0 = 47,045$  s and  $T_0 = 1,28$  s)**

The product of  $h_w(t)$  and  $v_z(t)$  is transformed into a frequency spectrum using a discrete Fourier transformation. The spectrum is shown in figure D.4 as a normalized spectrum where the maximum of  $f_z$  is 16,5 Hz.



**Figure D.4: Normalized spectrum**

The value given in line 2 of table 1 for a dwelling and a frequency of 16,5 Hz is 6,6 mm/s. The measured maximum of 5,1 mm/s lies beneath this value.